ANU GRADUATE PROGRAM IN ECOLOGY EVOLUTION AND SYSTEMATICS
MODELLING WORKSHOP

SOME BASIC THEORETICAL POPULATION MODELS

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EXERCISES

DENSITY-INDEPENDENT POPULATION GROWTH

Geometric Growth Model

Basic exercise

Run the model with a few different values of R above and below the value 1. What do you conclude? Try some values of R very near 1. Note that growth is then very nearly linear, at least for a few intervals of time. Work out the formula for this approximately linear growth.

More complex exercise

Plot $N(t)$ on a log scale so that you can assure yourself that it is a straight line.

Geometric Growth in a Temporally Variable Environment

Basic exercise

Check the claim that growth is approximately exponential at the exponential rate μ. Try gradually increasing the variance. How does this affect population growth?

More complex exercise

Plot the population on a log scale and on the same graph plot the asymptotic approximation. How good is this asymptotic approximation? Can it reliably predict when growth will be positive in the "long run?" What factors affect the reliability of this prediction?

Express the finite rate of increase as a sum of adult survival and adult recruitment rates. Further subdivide adult recruitment into the birth rate and juvenile survival rate. How do fluctuations in these various rates affect overall fluctuations in $\ln R(t)$? Thinking about the life-histories of various real organisms, which of these parameters would you expect to be most variable in real organisms?

DENSITY-DEPENDENT GROWTH

Ricker Model

Basic exercise

Gradually increase the value of R from 2 to 20. How do the dynamics of the system change? How does the equilibrium point change? How does α affect population fluctuations? How does it affect the equilibrium point?

More complex exercises

Show that even when fluctuating chaotically, a Ricker population fluctuates between fixed upper and lower limits.

Derive a the linear approximation to the dynamical equation and use it to derive the conditions for linear stability.

Prove, using analytical mathematics, that the time average of population size is equal to K in the long run. *(Hint: calculate the long-term growth rate and use the fact that this growth rate is zero. Why is it zero?)* This result that the long-run average is equal to the equilibrium point is a special feature of this model. What is it about this model that is responsible for this result?

Plot $N(t+1)$ as a function of $N(t)$ in this model. What does the shape of this curve imply about the nature of density dependence in the species being modelled?

HOST-PARASITOID SYSTEMS

Nicholson-Bailey Model

Basic exercises

Examine the dynamics of this system first of all as a function of the distance of the initial values from equilibrium. How do the parameters R and a affect the dynamics of the system? Note that as you change parameters you should keep the initial value the same proportionate distance from equilibrium in order to have a useful comparison.

More complex exercises

Derive dynamical equations for $M = aN$ and $Q = aP$. What interpretation can you put on M and Q?

Can a model that behaves like this one possibly describe nature? And if not, what is its value?

May's Negative Binomial Host-Parasitoid Model

Basic exercises

Examine the dynamics of this system first of all as a function of the k. How do the parameters R and a affect the dynamics of the system? Note that as you change parameters you should keep the initial value the same proportionate distance from equilibrium.

How does k affect stability? How does k affect the equilibrium?

More complex exercise

Compare host survival rates as a function of parasitoid density in the Negative binomial and Nicholson-Bailey models. Plotting log (host survival) against P should give the most helpful comparison. Use your results to formulate an explanation of how small values of k stabilise the host-parasitoid interaction.

Lottery Competition with Temporally Variable Birth Rates

Basic exercises

First investigate the model with zero values for σ_i^2 and σ_j^2. Determine the differences in the μs and δs that lead to competitive exclusion in 250 and 500 time units. With these particular values of the μs and δs, gradually increase the σ^2 values until you are confident that long-term coexistence has been achieved. (How can you be confident that they are coexisting if you cannot increase the length of the simulation?) Compare your results with the approximate formula for $\overline{r}_i$.

Using your results above, determine a combination of parameter values that leads to a nice long-term stable coexistence. Gradually increase the values of the δs while maintaining a constant ratio for these two deltas. How is coexistence affected?

More complex exercises

Write a spread-sheet program to calculate the long-term low-density growth rate, and compare your results with the quadratic approximation formula that you have been given.

Rewrite the model that you have been given so that newly settled individuals are subject to stochastic mortality. Can you see a justification for a model of this sort? What is the effect of variance in this post-settlement mortality?

It is often implied that environmental variation promotes species coexistence by reducing population densities levels to densities where competition no longer occurs. Is that how coexistence occurs in the lottery model? Why do small adult death rates favour coexistence in the lottery model?

The model of geometric growth in a variable environment and the lottery model, as implemented here, both use log normal temporal variation in the parameters. Increasing the parameter σ^2 of the lognormal leads quickly to violent population fluctuations in the geometric growth model, but can lead to relative stability in the lottery model. Why does this difference exist? Frame your discussion in terms of the biological assumptions made in the two models?

Lottery Competition with Temporally Variable Adult Death Rates

Basic exercise

Unlike the other models that you have been given, the outcome of this model (who survives and who goes extinct) is stochastic. Try a few runs of the model to assure yourself that the outcome is stochastic. How is this conclusion affected by symmetry of the parameters of the two species?

How is speed of extinction affected by the death rate variance?

More complex exercise

Write a program to estimate the long-term low density-growth rate in this model, and use your program to determine the relationship between $\bar{r}_i$ and $\ln(B_i/B_j)$ for fixed values of the other parameters.

Write a program to estimate the frequency with which a particular species goes extinct in replicate runs of the model, and use your results to see how this frequency relates to $\bar{r}_i$ calculated from the other program that you have written.

Compare the two different formulations of the lottery given here. Which is more realistic and why?