Carbon Accounting and Averaging

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Background

"Scaling up" is a commonly used phrase in carbon accounting and throughout the ecological sciences. By the term, people usually mean that measurements are made at local scales (e.g. leaves, plants), but estimates are required at larger scales, e.g. forests stands, continents, global. This disparity between what is (traditionally) measured and what is needed (i.e. stand to continental scale estimates) is often referred to as the "scaling problem".

There is little doubt that there are significant practical difficulties in making suitable measurements; however, there are also some theoretical considerations. This short note describes one of the significant problems – the problem of averaging in the process of scaling up. Mathematical formulations typically incorporate products and quotients of system variables, and the problem of averaging occurs when averages of these terms are taken. The basis of the problem is that the average of a product (what we usually want) is not necessarily the same as the product of the averages (what is often calculated), and the difference between the two may be significant. Similarly, for quotients, the average of a quotient is not necessarily equal to the quotient of the averages.

In this note we provide an example, based on a fundamental task in terrestrial carbon accounting - estimating the carbon storage in forest trees, to illustrate the importance of this issue.

The Problem of Averages in Carbon Accounting

Assume that for a stand of trees we require an estimate of the total carbon. For each individual tree, we can express the carbon (C_i, kg) as,

$$C_i = \frac{C_i}{m_{d,i}} \frac{m_{d,i}}{V_i} V_i = C_{di} \frac{m_{d,i}}{V_i} V_i \tag{1}$$

where $m_{d,i}$ (kg) is the dry mass, V_i (m^3) is the volume and C_{di} (= $C_i / m_{d,i}$) is the mass fraction of the dry matter which is carbon, for the individual. In practical applications C_{di} is usually set at a constant value of about 0.5. (Note that it would be 0.44 if all the dry matter in a tree was cellulose.) The ratio $m_{d,i}/V_i$ is known as the basic density in the forestry literature and denoted here as $[D_i]$, so that, from above, we have,

$$C_i = C_{di}\,[D_i]\,V_i\,. \tag{2}$$

In most practical applications V_i is known (through an allometric relationship, for example) and we require an estimate of C_i.

Now, to scale Eqn 1 to the level of a stand comprised of n individual trees,

$$\sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left(C_{di} \, [D_i] V_i \right) \approx C_d \sum_{i=1}^{n} \left([D_i] V_i \right) = C_d \; n \; \overline{[D_i] V} \qquad (3)$$

The mathematics seems straightforward. However, the crux of the matter is that, in practice, Eqn 3 is commonly calculated using the product of the averages, $\overline{[D_i]}\,\overline{V_i}$, while what is required is the average of the product, $\overline{[D_i] V_i}$. We distinguish between these by noting that the average of the product is equal to the product of the averages plus the covariance, that is,

$$\overline{[D_i] V_i} = \overline{[D_i]}\,\overline{V_i} + \sigma_{[D]V} \qquad (4)$$

where $\sigma_{[D]V}$ is the covariance between the basic density and volume across individuals within the stand. By definition, the covariance is given by the product of the correlation coefficient ($r_{[D]V}$) and the standard deviations ($\sigma_{[D]}, \sigma_V$). In terms of the more convenient coefficients of variation ($\varepsilon_{[D]} = \sigma_{[D]} / \overline{[D]}, \varepsilon_V = \sigma_V / \overline{V}$), Eqn 4 becomes,

$$\overline{[D_i] V_i} = \overline{[D]}\,\overline{V} + r_{[D]V} \varepsilon_{[d]} \overline{[D]} \, \varepsilon_V \overline{V} = \overline{[D]}\,\overline{V}(1+S) \qquad (5)$$

where S ($= r_{[D]V} \varepsilon_{[d]} \; \varepsilon_V$) is called here the scaling correction. Substituting back into Eqn 3 we have that,

$$\sum_{i=1}^{n} C_i \approx C_d \; n \, \overline{[D]}\,\overline{V} \, (1+S) = C_d \; \overline{[D]} \, (1+S) \sum V \, , \qquad (6)$$

where $\sum V$ is total wood volume usually available from forest inventory estimates. The basic density can be estimated by measurement, or taken from the literature. It typically lies between 0.1 to 1.1 g cm^{-3}.

The key here is that Eqn 6 demonstrates that the average of the product (what is required for an accurate calculation of total carbon) will only equal the product of the averages (what is often calculated) when S is zero.

Several years ago, we put together an example to explain the significance of this result. The attached spreadsheet provides this worked example, illustrating the impact of neglecting S in estimates of stand scale carbon.

Postscript

Roderick and Berry (2001) proposed a generic framework to help estimate how [D] changes with environment (and climate change). The ultimate aim of that research was to use the theory to predict how C storage in forests would change in future. Since then, the theory has been tested quantitatively using both field measurements (Atwell *et al.*, 2003; Barbour and Whitehead, 2003) and experimental treatments (Thomas *et al.*, 2004) and found to be in accord with observations. This means that we can apply that research within the above-noted framework to estimate how C storage of forests might change in future.

Further Reading

Also see Barnes and Roderick (2004) for a more detailed account.

References

Atwell BJ, Henery ML, Whitehead D. 2003. Sapwood development in *Pinus radiata* trees grown for three years at ambient and elevated carbon dioxide partial pressures. *Tree Physiology* **23**: 13-21.

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